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Hidden extreme multistability in memristive hyperchaotic system

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ABSTRACT

By utilizing a memristor to substitute a coupling resistor in the realization circuit of a three-dimensional chaotic system having one saddle and two stable node-foci, a novel memristive hyperchaotic system with coexisting infinitely many hidden attractors is presented. The memristive system does not display any equilibrium, but can exhibit hyperchaotic, chaotic, and periodic dynamics as well as transient hyperchaos. In particular, the phenomenon of extreme multistability with hidden oscillation is revealed and the coexistence of infinitely many hidden attractors is observed. The results illustrate that the long term dynamical behavior closely depends on the memristor initial condition thus leading to the emergence of hidden extreme multistability in the memristive hyperchaotic system. Additionally, hardware experiments and PSIM circuit simulations are performed to verify numerical simulations.

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1. Introduction

Conventionally, a nonlinear dynamical system described by a set of autonomous ordinary differential equations can be easily implemented with an electronic circuit via standard op-amp integrators and analogue multipliers, where an op-amp integrator linked with a feedback capacitor and some inverting input resistors performs the mathematical operation of integration with respect to time, and a functionally complete four-quadrant analogue multiplier achieves the mathematical operation of nonlinearity [1,2]. Memristor [3], a fundamental two-terminal electronic element with an adjustable resistance or conductance [4], has potential applications in various nonlinear dynamical circuits due to the special features of both nonlinearity and memory. In recent years, by introducing memristors into existing oscillating circuits or substituting nonlinear resistors in classical chaotic circuits with memristors, a variety of memristor based chaotic/hyperchaotic circuits are simply established and broadly investigated [1,5–7]. In this paper, based on the realization circuit of a three-dimensional chaotic system, a systematic circuit realization scheme of a four-dimensional memristor based hyperchaotic system is proposed by utilizing a memristor to substitute a linear coupling resistor.

The conventional self-excited attractors, such as Lorenz attractor [8], Chen attractor [9], Lü attractor [10], and many other widely-known attractors [11–14], are all excited from unstable index-2 saddle-foci, namely, an attractor with an attraction basin corresponds to an unstable equilibrium. Consequently, self-excited attractors can easily be implemented by disposing unstable index-2 saddle-foci in terms of added breakpoints in the model system [15,16]. However, a new type of attractors, defined as hidden attractors [17], have been first found in classical Chua's circuit [18,19], whose basin of attraction does not intersect with small neighborhoods of the equilibria of the system [17]. Due to the existences of hidden attractors, some particular dynamical systems associated with line equilibrium, or no equilibrium, or stable equilibrium have attracted much attention recently [20–26]. In particular, to the extent that they have been known to exist, dynamical systems with no equilibrium have mostly been considered as unphysical or mathematically incomplete. However, as experience shows, a system that presents hidden dynamical behavior doesn't need to also display an unstable equilibrium state [23–26]. In this paper, a memristor based hyperchaotic system with no equilibrium is proposed, from which the complex and striking phenomenon of coexisting infinitely many hidden attractors' behavior and the corresponding hidden extreme multistability are perfectly revealed. To the best knowledge of the authors, this phenomenon of coexisting infinitely many hidden attractors has not been reported in any literatures.

Multistability, meaning the coexistence of many different kinds of attractors, is an intrinsic property of many nonlinear dynamical systems and has become very important research topic and received much attention recently [27–32]. Multistability exhibits a rich diversity of stable states of a nonlinear dynamical system and makes the system offer a great flexibility. Particularly, when the number of coexisting attractors generating from a dynamical system tends to infinite, the coexistence of infinitely many attractors depending on the initial condition of a certain state variable is alleged extreme multistability [33], which has been reported in two

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unidirectionally coupled Lorenz systems [34], two bi-directionally coupled Rössler oscillator with partial synchronization [33,35], and several memristive circuit with ideal active flux-controlled memristors [36–38]. Since multistability can be used for image processing [39] or regarded as an additional source of randomness using for many information engineering applications [32], it is attractive to seek for a memristor based hyperchaotic system that has the striking dynamical behavior of infinitely many attractors. Actually, in a memristor based chaotic circuit [40,41] or a chaotic memory system [2], the complex dynamical behaviors are dependent on the initial states of the memristor or the memory element, which just reflect the emergences of extreme multistability in these memory systems.

In this paper, by utilizing a newly proposed circuit realization scheme, a novel memristive hyperchaotic system is derived from an existing three-dimensional chaotic system [42], from which the coexisting phenomenon of infinitely many hidden attractors, i.e., hidden extreme multistability, is discovered and then revealed. It is important to note, that different from extreme multistability emerged from line equilibrium reported in Refs. [36–38], the infinitely many hidden attractors found in this paper are not related to any equilibrium. The paper is organized as follows. In Section 2, a circuit realization scheme is proposed and a memristive hyperchaotic system with no equilibrium is then constructed. In Section 3, with system parameters varying, bifurcation diagrams and the corresponding Lyapunov exponent spectra are plotted to reveal hidden hyperchaotic dynamical behaviors. Additionally, transient hyperchaotic behavior is also illustrated. In Section 4, with memristor initial condition varying, bifurcation diagram, Lyapunov exponent spectra, and phase portraits are given, upon which infinitely many hidden attractors' dynamics and transient transition behavior are demonstrated. Experimental measurements and PSIM simulations are performed in Section 5 to validate numerical simulations. The conclusions are summarized in Section 6.

2. Memristive hyperchaotic system with hidden attractor

A circuit realization scheme of four-dimensional memristive hyperchaotic system is proposed in this section, which is achieved by utilizing a memristor to substitute a self-variable resistor or linear coupling resistor in the realization circuit of an existing three-dimensional chaotic system.

2.1. Circuit realization scheme

A three-dimensional chaotic system presented by Ref. [42] is an interesting dynamical system with one saddle and two stable node-foci, which is described as

$$\begin{cases} \dot{x} = ay - ax \\ \dot{y} = cx - xz \\ \dot{z} = xy - bz \end{cases} \quad (1)$$

where a, b, and c are real positive parameters. When parameters $a=35$, $b=3$, $c=35$, system (1) has one unstable zero equilibrium (0, 0, 0) and two stable nonzero equilibria $(\pm\sqrt{105}, \pm\sqrt{105}, 35)$. The eigenvalues of the linearized system evaluated at the unstable zero equilibrium are: $\lambda_1=-3$, $\lambda_2=-56.6312$, $\lambda_3=21.6312$; whereas the eigenvalues at the stable nonzero equilibria are: $\lambda_1=-37.6122$, $\lambda_{2,3}=-0.1939\pm j13.9778$. With initial conditions (1.15, 3.5, 3.3), the Lyapunov exponents of system (1) are: $L_1=1.0742$, $L_2=0$, $L_3=-39.074$ and the corresponding fractional dimension is $D_L=2.0275$. Consequently, system (1) generates a double-scroll chaotic attractor coexisting with one saddle and two stable node-foci. The analytical and numerical results for system (1) can be found in Ref. [42].

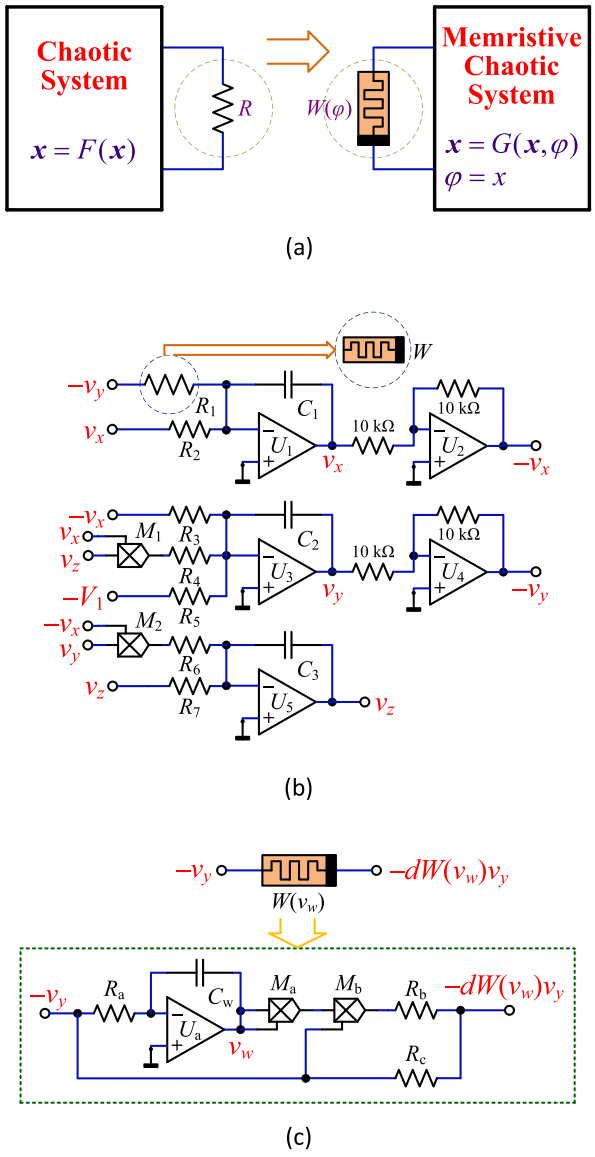

Fig. 1. Circuit realization of the proposed memristive hyperchaotic system: (a) circuit realization scheme by utilizing a memristor W to substitute a resistor R, (b) circuit realizations of the chaotic system (1) and the memristive hyperchaotic system (6) constructed by replacing R_1 with W, (c) circuit realization of the flux-controlled memristor.

Just like as the famous Lorenz system [8], Chen system [9], and Lü system [10], system (1) is also a simple three-dimensional autonomous systems with two quadratic nonlinearity terms, which is easily implemented with an electronic circuit via multipliers and op-amps linked with resistors and/or capacitors. Based on the realization circuit of system (1) and by utilizing a memristor to substitute a self-variable resistor or linear coupling resistor, a novel memristive chaotic system is conveniently constructed, which can be intuitively described as shown in Fig. 1(a), where R stands for a self-variable resistor or linear coupling resistor and W represents a memristor. This thinking is similar to that presented in [31] but different from that reported in [1,43,44]. Note that the four-dimensional memristive hyperchaotic systems in [1,43,44] are derived from some original three-dimensional systems through introducing memristors.

The realization circuit of system (1) is displayed in Fig. 1(b), in which there are three voltage state variables v_x, v_y, and v_z, respectively corresponding to three capacitors with same capacitance C.

Applying Kirchhoff's circuit laws and the constitutive relations, a state equation set is yielded as

$$\begin{cases} C\dot{v}_x = v_y/R_1 - v_x/R_2 \\ C\dot{v}_y = v_x/R_3 - v_x v_z/R_4 + V_1/R_5 \\ C\dot{v}_z = v_x v_y/R_6 - v_z/R_7 \end{cases} \tag{2}$$

where the nonzero constant term V_1/R_5 is newly introduced, which can make the next proposed memristive hyperchaotic system have no equilibrium. Therefore, the nonzero constant term is especially valuable for the emergence of hidden extreme multistability in the proposed system.

Considering that RC is the time constant of the three integrators, t is the dimensionless time, and $t \bullet RC$ is the physical time, the parameters can then be taken as follows: $a = R/R_1 = R/R_2$, $b = R/R_7$ and $c = R/R_3$. Note that the gains of two multipliers M_1 and M_2 are set to one, $R_4 = 0.05R$ and $R_6 = 0.05R$ are selected to reduce the amplitude of every voltage variable, and then the constant term $\mu = 20RV_1/R_5$ can be determined.

According to the circuit realization scheme of memristive chaotic system in Fig. 1(a), a flux-controlled memristor W is used to replace the resistor R_1 in Fig. 1(b). Thus, a memristor-based dynamical system is easily built. For the memristor W, the nonlinear constitutive relation between the terminal voltage v and terminal current i is given by

$$i = W(\varphi)v, \dot{\varphi} = v \tag{3}$$

where $W(\varphi)$ is a memductance function, describing the flux-dependent rate of change of charge. Like many other literatures [1,44,45], the following memductance function is considered in our next work.

$$W(\varphi) = \alpha + \beta\varphi^2 \tag{4}$$

where α and β are two positive constant parameters.

With (4), an op-amp and multiplier based realization circuit of the flux-controlled memristor W is shown in Fig. 1(c), where the time constant RC of the integrator is consistent with those in Fig. 1(b), the parameters d, α and β are satisfied the relationships of $R_a = R$, $d\alpha = R/R_c$, and $d\beta = 0.0025R/R_b$ (the coefficient 0.0025 is considered to reduce the amplitude of voltage variable with similar scale factor). Therefore, the state equation set of the circuit of the newly constructed memristive system can be written by

$$\begin{cases} C\dot{v}_x = (v_y/R_c + v_w^2 v_y/R_b) - v_x/R_2 \\ C\dot{v}_y = v_x/R_3 - v_x v_z/R_4 + V_1/R_5 \\ C\dot{v}_z = v_x v_y/R_6 - v_z/R_7 \\ C\dot{v}_w = v_y/R_a \end{cases} \tag{5}$$

Correspondingly, a dimensionless state equation set of the proposed memristive system is modeled as

$$\begin{cases} \dot{x} = dW(w)y - ax \\ \dot{y} = cx - xz + \mu \\ \dot{z} = xy - bz \\ \dot{w} = y \end{cases} \tag{6}$$

where, $W(w) = \alpha + \beta w^2$, the newly adding parameter d is a positive parameter indicating the strength of the memristor, and μ is a nonzero control constant.

Observed from Fig. 1(b), it can be seen that R_1 and R_3 are two coupling resistors, whereas R_2 and R_7 are two self-variable resistors. The realization circuit of the memristive system only constructed by replacing R_1 or R_2 in Fig. 1(b) with W can generate chaos or hyperchaos. However, if the newly realized circuit is obtained by replacing R_1 with W, the circuit can not generate hyperchaos and its dissipativity can not maintain unchanged. Therefore, the proposed system (5) is the most appropriate memristive hyperchaotic system which is derived from system (2).

The memristive hyperchaotic system (6) is a four-dimensional smooth nonlinear dynamical system. Based on (6), some interesting dynamical behaviors can be observed and analyzed by numerical simulations.

2.2. Typical hidden hyperchaotic attractor

The new four-dimensional system (6) has the same dissipativity as the original three-dimensional system (1). Obviously, the memristive hyperchaotic system (6) is asymmetrical due to the introduction of the constant term μ.

By setting the left-hand side to zero, one has

$$\begin{cases} 0 = dW(w)y - ax \\ 0 = cx - xz + \mu \\ 0 = xy - bz \\ 0 = y \end{cases} \tag{7}$$

When $\mu \neq 0$, (7) has no real solutions and system (6) thus has no equilibrium, implying that when μ is nonzero, the generating attractors of system (6) are all hidden, whose basin of attraction that does not intersect with small neighborhood of any equilibrium [21,23,25]. Furthermore, the terms on the left-hand side of (7) must time-average to zero for the attractors to be bounded [21].

When the typical parameters are fixed as $a = 35$, $b = 3$, $c = 35$, $d = 40$, $\mu = 1$, $\alpha = 1$, $\beta = 0.02$, and the initial conditions are chosen as (0.1, 0, 0, 0), the memristive system (6) exhibits a double-scroll hyperchaotic attractor, as shown in Fig. 2, from which it can be seen that the system has topologically more complex attractor structure than system (1) presented by [42]. The corresponding Lyapunov exponents are $L_1 = 0.5881$, $L_2 = 0.1306$, $L_3 = 0$, $L_4 = -37.7922$ and the Kaplan–Yorke fractional dimension is $D_L = 3.0019$. The Poincaré mappings for the memristive system (6) on two sections $x = y$ and $z = 40$ are depicted in Fig. 3. It can be found from Fig. 3 that the Poincaré images have no regular limbs, implying that the system has extremely rich dynamics and it is different from the normal chaotic systems with one or more equilibria [25].

Consequently, the newly constructed memristive system has no equilibria but is hyperchaotic, i.e., the memristive system operates in hidden hyperchaotic oscillation.

3. Hidden hyperchaotic behavior depending on parameters

It is well known that the main dynamical property of the memristive system (6) can be analyzed through bifurcation diagrams and the corresponding Lyapunov exponent spectra. To investigate the effect of the parameters on the dynamics of the proposed memristive hyperchaotic system, the parameters $a = 35$, $b = 3$, $\mu = 1$, $\alpha = 1$, $\beta = 0.02$ as well as the initial conditions (0.1, 0, 0, 0) are fixed and the parameters c and d are respectively adjusted.

3.1. Hidden bifurcation behavior

When $c = 35$ and d is gradually increased in the range of 5 to 50, the bifurcation diagram of the state variable y and the four Lyapunov exponents calculated by Wolf's method are plotted in Fig. 4(a) and (b), respectively. Similarly, when $d = 40$ and c is gradually increased in the range of 20 to 60, the bifurcation diagram of the state variable y and the four Lyapunov exponents are plotted in Fig. 4(c) and (d), respectively.

From Fig. 4, the dynamical behaviors of chaos with one positive Lyapunov exponent and hyperchaos with two positive Lyapunov exponents are observed, and periodic windows with zero largest Lyapunov exponent and reverse period-doubling routes to chaos in

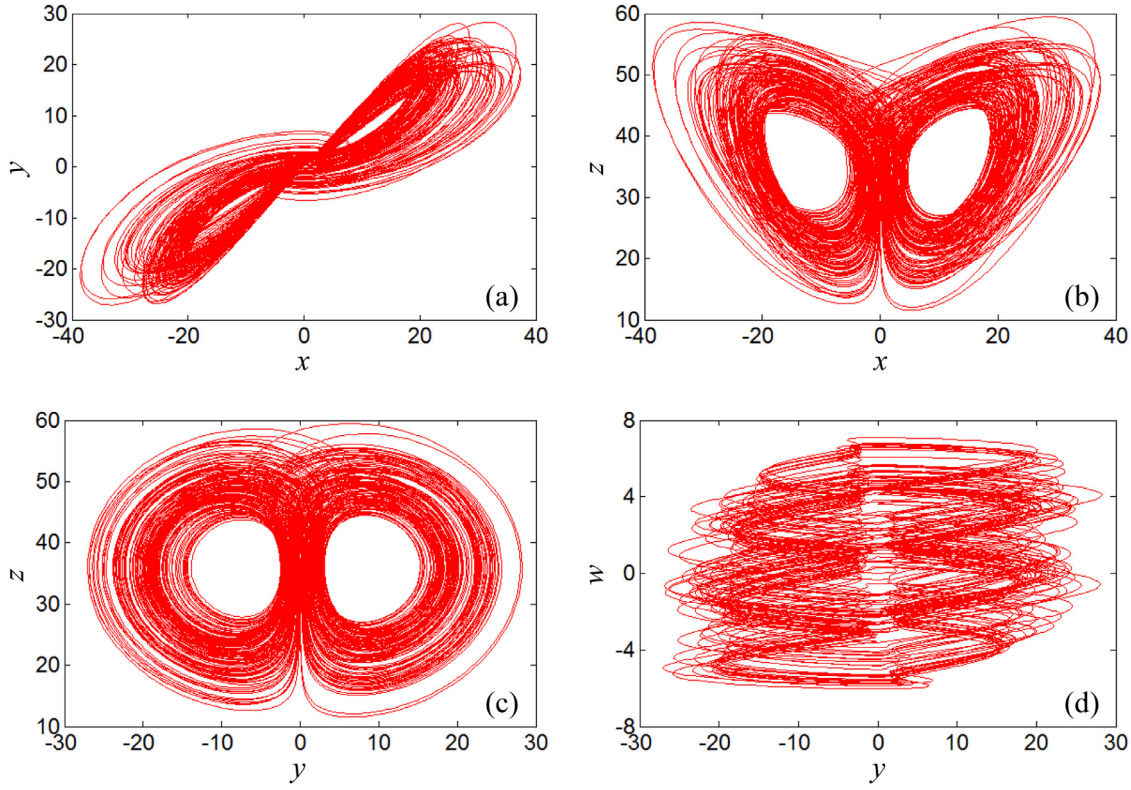

Fig. 2. Different perspectives on the hyperchaotic attractor of the memristive hyperchaotic system (6) with no equilibria, $a=35$, $b=3$, $c=35$, $d=40$, $\mu=1$, $\alpha=1$ and $\beta=0.02$.

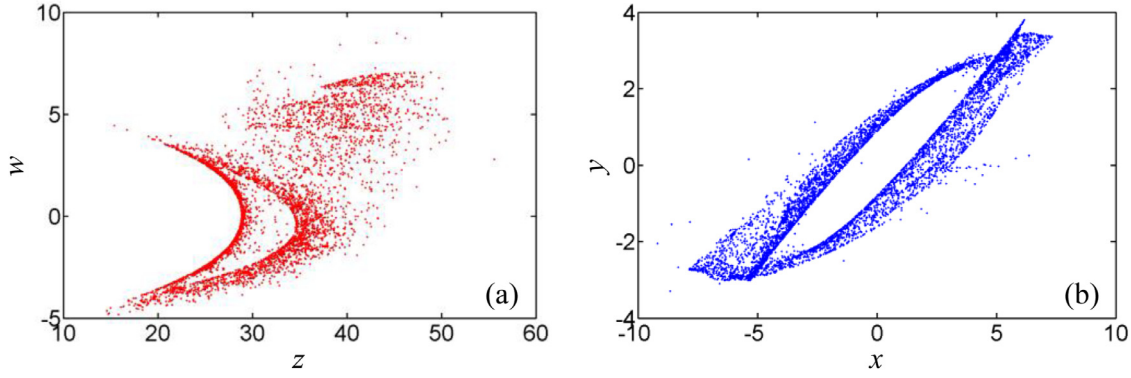

Fig. 3. Poincaré maps of the hyperchaotic attractor of the memristive system (6) with $a=35$, $b=3$, $c=35$, $d=40$, $\mu=1$, $\alpha=1$ and $\beta=0.02$ on crossing sections (a) $x=y$ and (b) $z=40$.

periodic windows are also found in the hyperchaotic regions. Furthermore, the results in Fig. 4 show that the memristive system (6) indeed generate hyperchaotic attractors for a wide range of parameters c and d. Note that when the parameter d is increased, the dynamic amplitude of the state variable y is reduced, while when the parameter c is increased, the dynamic amplitude of the state variable y is enlarged.

3.2. Transient hyperchaotic behavior

Transient chaos is a common phenomenon observed in many nonlinear dynamical systems [6,7,46], wherein an orbit behaves chaotically for a finite time interval before settling into a final non-chaotic state. In contrast to the transient chaos, transient hyperchaos, a complex nonlinear phenomenon with two positive Lyapunov exponents in the infinite time interval, has been found in the memristive hyperchaotic systems [6,7].

When $d=36$, a phenomenon of transient hyperchaos is generated in the proposed memristive hyperchaotic system under the initial values (0.1, 0, 0, 0), as shown in Fig. 5, where the trajectories of the system have a transition from transient hyperchaotic to steady periodic behaviors with the time evolutions. Note that these phase portraits in Fig. 5(a) and (b) are hidden attractors, the time-domain waveform in Fig. 5(c) oscillates in hidden pattern, and the maximum Lyapunov exponent L_1 in Fig. 5(d) tends to zero on an infinite time scale [41].

4. Extreme multistability relying on memristor initial condition

In this section, hidden dynamical behaviors closely relying on the memristor initial condition are investigated. To some extent, memristor initial condition dependent hidden dynamics just reveals the extreme multistability phenomenon of coexisting in-

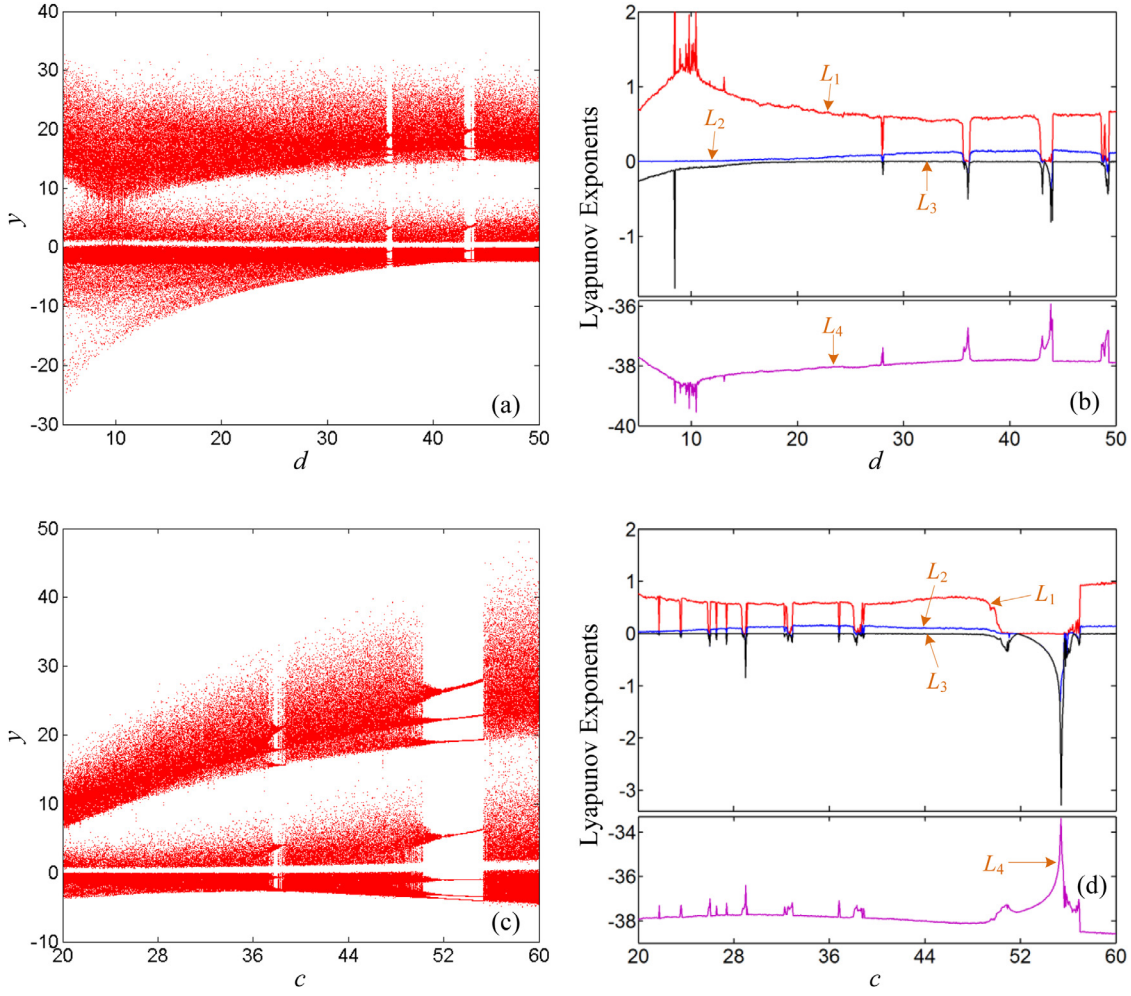

Fig. 4. Hidden hyperchaotic dynamics: (a) bifurcation diagram of the state variable y and (b) Lyapunov exponents with the variation of parameter d, (c) bifurcation diagram of the state variable y and (d) Lyapunov exponents with the variation of parameter c.

finitely many hidden attractors' behavior in the memristive system (6).

4.1. Coexisting infinitely many hidden attractors dynamics

The typical parameters $a=35$, $b=3$, $c=35$, $d=40$, $\mu=1$, $\alpha=1$, $\beta=0.02$ and three initial conditions $x(0)=0.1$, $y(0)=0$, $z(0)=0$ are determined, whereas the memristor initial condition $w(0)$ is varied in the region $[-80, 80]$. When the memristor initial condition $w(0)$ is gradually increased from -80 to 80, the bifurcation diagram of the state variable y and the four Lyapunov exponents are plotted in Fig. 6(a) and (b), respectively. From Fig. 6, it is found that the memristive system (6) exhibits hyperchaotic, chaotic, periodic, and intermittent chaotic dynamics with the variation of the memristor initial condition. The hyperchaotic behavior with two positive Lyapunov exponents is mainly located in the region $[-36, 32]$. Meanwhile, several narrow chaotic windows with one positive Lyapunov exponent in the hyperchaotic region and some periodic windows with zero largest Lyapunov exponent in the chaotic region are also found.

It should be specially remarked that the infinite number of coexisting hidden attractors is in fact confirmed by Fig. 6. According to Fig. 6, several positive and negative memristor initial conditions are considered. When $w(0)$ is set to 10, 25, 40, 49, 70, respectively, the phase portraits of coexisting hidden multiple attractors in the x-z plane and the w-z-x space are displayed in Fig. 7(a) and (b). While when $w(0)$ is set to -20, -30, -35, -50, -65, respectively, the phase portraits of coexisting hidden multiple attractors in the x-z plane and the w-z-x space are displayed in Fig. 7(c) and (d). Fig. 7 clearly reveals the coexistence of a large number of hidden attractors with different dynamical behaviors, implying the emergence of extreme multistability with hidden oscillation in the memristive system (6).

Coexisting multiple attractors behavior is common in various nonlinear dynamical systems, including laser [47], Lorenz system [27], electrical circuits [29,32,48], chaotic system with only one stable node-focus [30], two bi-directionally coupled Rössler oscillator with partial synchronization [33,35], memristor-based circuits and systems [31,36–38], and so on. This behavior is the phenomenon of multiple stability involving the coexistence of three and more than three topologically different and disconnected attractors, or the phenomenon of extreme multistability associating with the coexistence of infinitely many different attractors. However, to the best knowledge of the authors, the phenomenon of coexisting infinitely many hidden attractors, i.e., hidden extreme multistability, shown in Fig. 7 has not been reported in any literatures.

4.2. Transient transition behavior

With some memristor initial condition $w(0)$, complex transient chaotic/hyperchaotic transition behaviors occur in the memristive

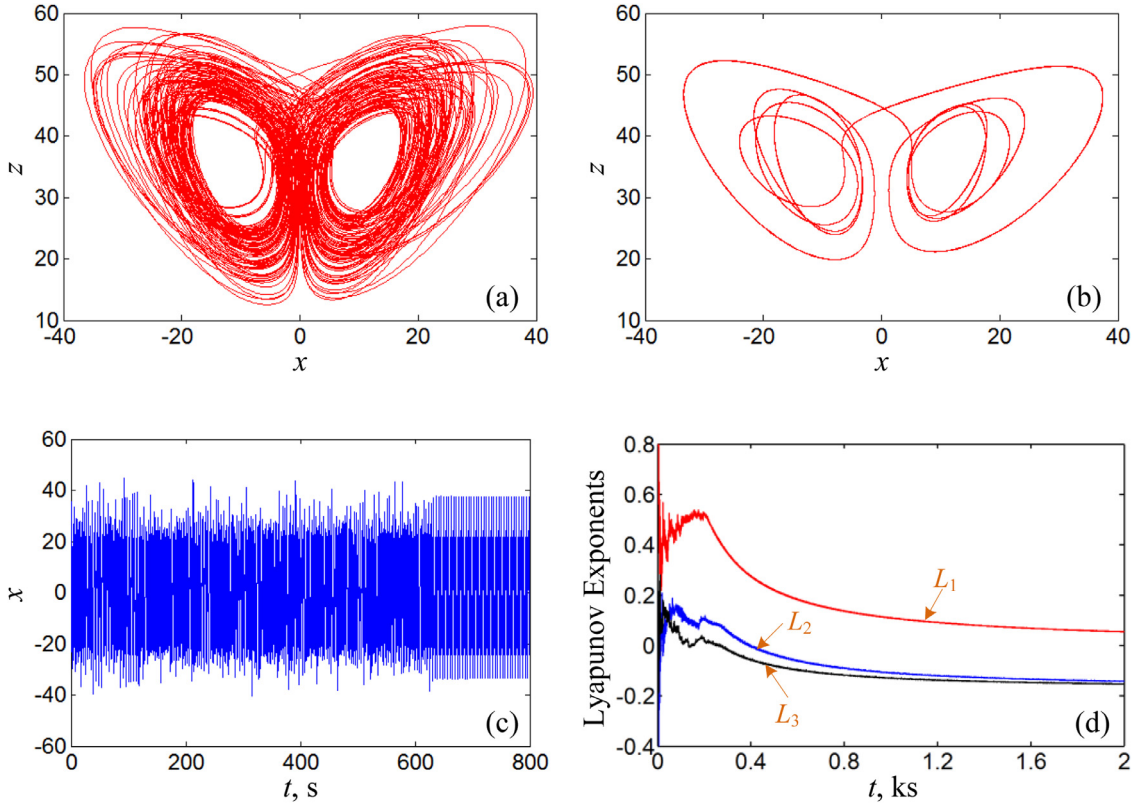

Fig. 5. Hidden transient hyperchaotic behavior: (a) projection on the x-z plane in the time interval [100 s, 200 s], (b) projection on the x-z plane in the time interval [700 s, 800 s], (c) time-domain waveform of the variable x, (d) first three Lyapunov exponents against time.

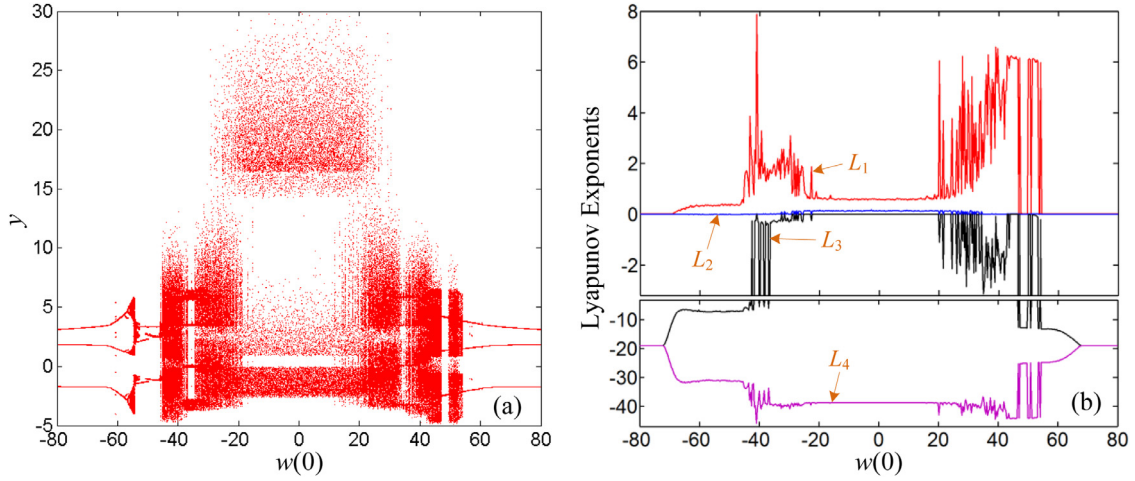

Fig. 6. Coexisting infinitely many hidden attractors' behavior with the variation of memristor initial condition $w(0)$: (a) bifurcation diagram of the state variable y, (b) Lyapunov exponent spectra.

system (6). For $w(0) = -15$, the trajectories of the system have a transition from a transient hyperchaotic attractor to another steady hyperchaotic attractor with two different dynamic amplitudes of the state variable x with the time evolutions. The phase portrait and time-domain waveform of the hyperchaotic attractor with transient transition behavior are shown in Fig. 8(a) and (b), where the Lyapunov exponents are $L_1 = 0.5905$, $L_2 = 0.1320$, $L_3 = 0$, $L_4 = -38.7055$. Whereas for $w(0) = 18$, the trajectories of the system have a transition from transient hyperchaotic to steady chaotic behaviors with the time evolutions. The phase portrait and time-domain waveform of the chaotic attractor with transient hyperchaos are shown in Fig. 8(c) and (d), where a hyperchaotic attractor is located in the time interval [0, 50 s] and then a chaotic attractor forms after $t > 50$ s. Correspondingly, the Lyapunov exponents are $L_1 = 3.6846$, $L_2 = 0$, $L_3 = -2.2236$, $L_4 = -39.4276$.

Additionally, intermittent chaotic behavior is easily found in the regions [−69, −46] and [50, 54] by comparing Fig. 6(a) with Fig. 6(b), which implies that there are periods of almost periodic interspersed with short chaotic bursts [49,50]. For initial condition $w(0) = -60$, the trajectories of the memristive system (6) converge to a chaotic attractor with intermittent periodic behavior, and the Lyapunov exponents are $L_1 = 0.3377$, $L_2 = 0$, $L_3 = -7.2332$, $L_4 = -31.0842$.

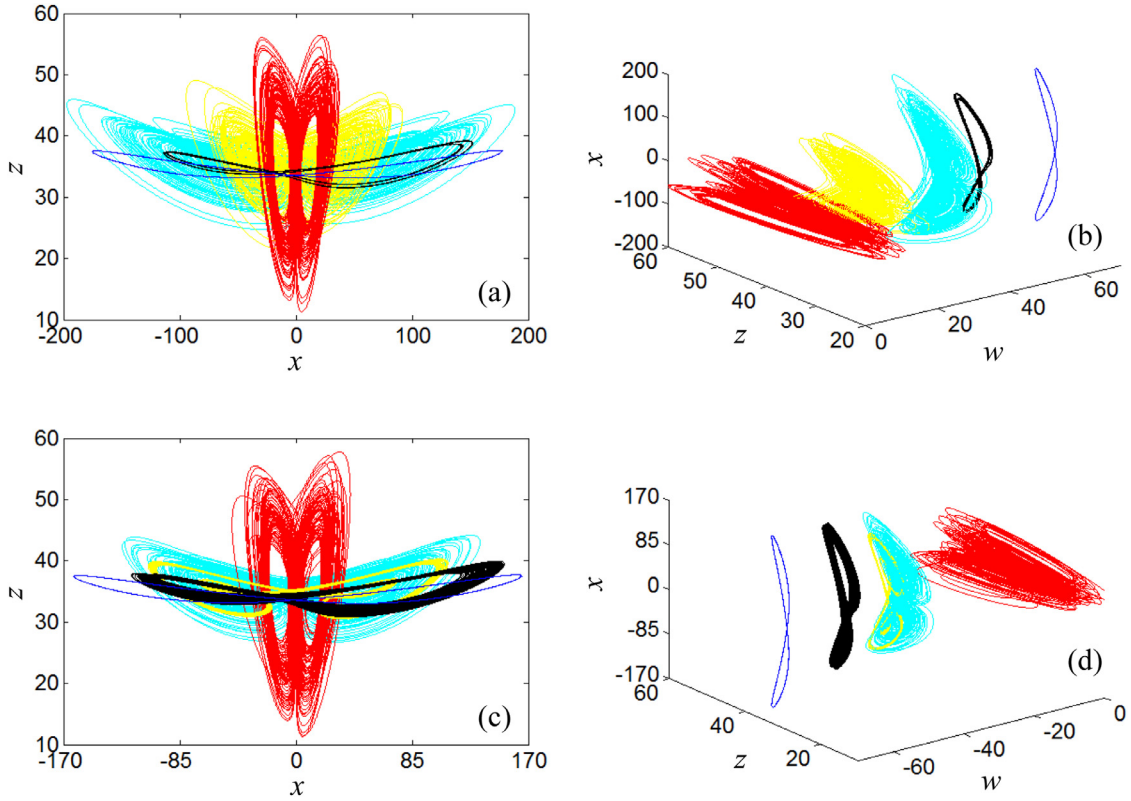

Fig. 7. Coexisting infinitely many hidden attractors in the memristive hyperchaotic system: projections of different attractors on the x-z plane and the w-z-x space for different memristor initial conditions, (a) the x-z plane and (b) the w-z-x space with several positive memristor initial conditions, (c) the x-z plane and (d) the w-z-x space with several negative memristor initial conditions. Where in (a) and (b), $w(0)=10$ (red), hyperchaos; $w(0)=25$ (yellow), hyperchaos; $w(0)=40$ (cyan), chaos; $w(0)=49$ (black), asymmetrical limit cycle; $w(0)=70$ (blue), symmetrical limit cycle. And where in (c) and (d), $w(0)=-20$ (red), hyperchaos; $w(0)=-30$ (cyan), chaos; $w(0)=-35$ (yellow), chaos; $w(0)=-50$ (black), intermittent chaos; $w(0)=-65$ (blue), symmetrical limit cycle. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

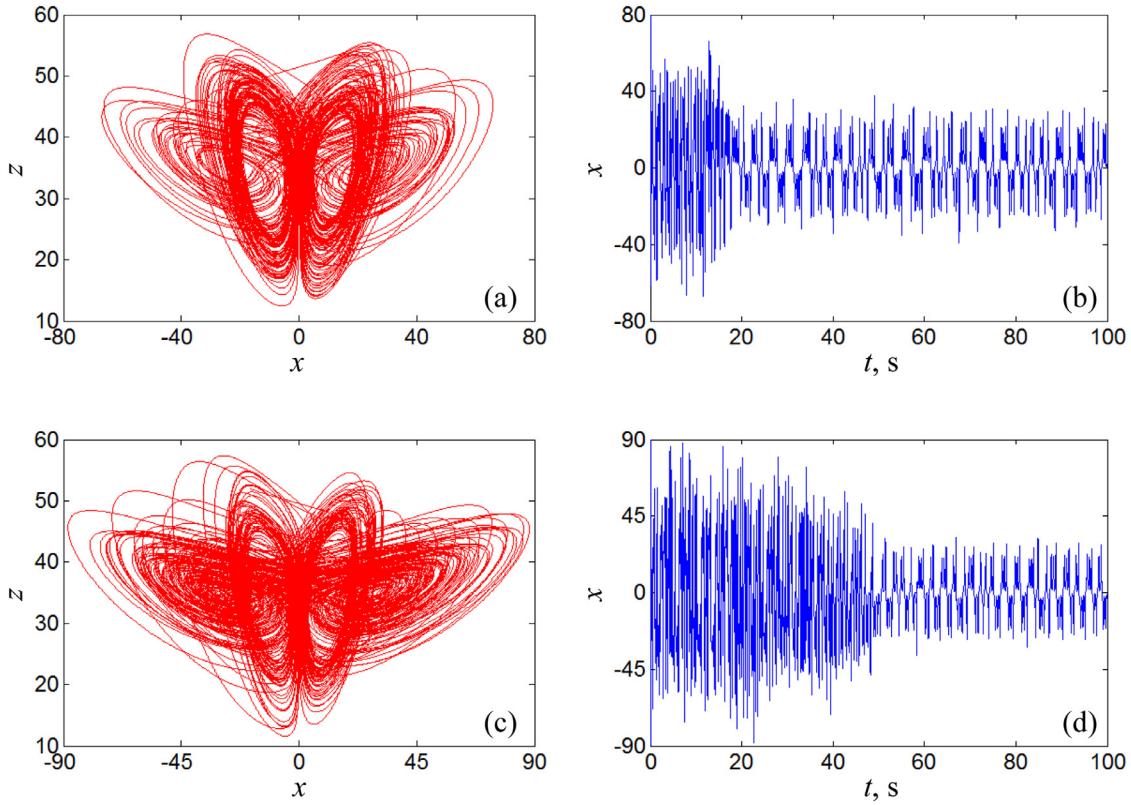

Fig. 8. Transient transition behaviors, (a) and (c) are phase portraits and (b) and (d) are time-domain waveforms, the memristor initial condition $w(0)$ of (a) and (b) equals to -15, whereas $w(0)$ of (c) and (d) is 18.

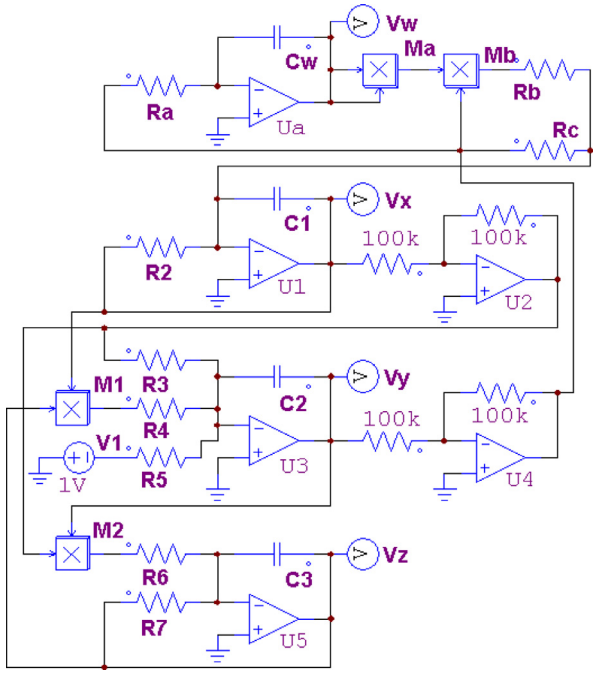

Fig. 9. PSIM circuit simulation model for the memristive hyperchaotic system.

5. Experimental measurements and PSIM simulations

Based on hardware experimental circuit and PSIM (Power SIMulation) circuit simulation model, the above hidden complex dynamics emerging from the proposed memristive hyperchaotic system can be demonstrated. Because the memristor initial condition with different desired values is hardly assigned in hardware experiments [19,48,51,52], the hidden dynamics of memristive hyperchaotic system depending on memristor initial condition is performed by PSIM circuit simulations. Note that PSIM circuit design and simulation software is an indispensable tool for power electronics and motor drives, which can also be used to simulate conventional electronic circuits.

By using PSIM Version 7.1 software, with the circuit realization of the memristive hyperchaotic system shown in Fig. 1(b) and (c), the circuit simulation model under typical circuit parameters is designed, as shown in Fig. 9. When the physical time $RC = 35$ ms and the capacitance $C = 100$ nF, the resistance is then determined as $R = 350$ kΩ. Therefore, for the typical parameters $a = 35$, $b = 3$, $c = 35$, $d = 40$, $\mu = 1$, $\alpha = 1$, and $\beta = 0.02$, the corresponding circuit parameters can be taken as follows: $R_a = R = 350$ kΩ, $R_b = 0.0025R/d\beta = 1.09$ kΩ, $R_c = R/d\alpha = 8.75$ kΩ, $R_2 = R/a = 10$ kΩ, $R_3 = R/c = 10$ kΩ, $R_4 = R_6 = 0.05R = 17.5$ kΩ, $R_5 = 20RV_1/\mu = 7$ MΩ, $R_7 = R/b = 116.7$ kΩ, $C_1 = C_2 = C_3 = C_w = C = 100$ nF, and the gains of two multipliers M_1 and M_2 are set to 1.

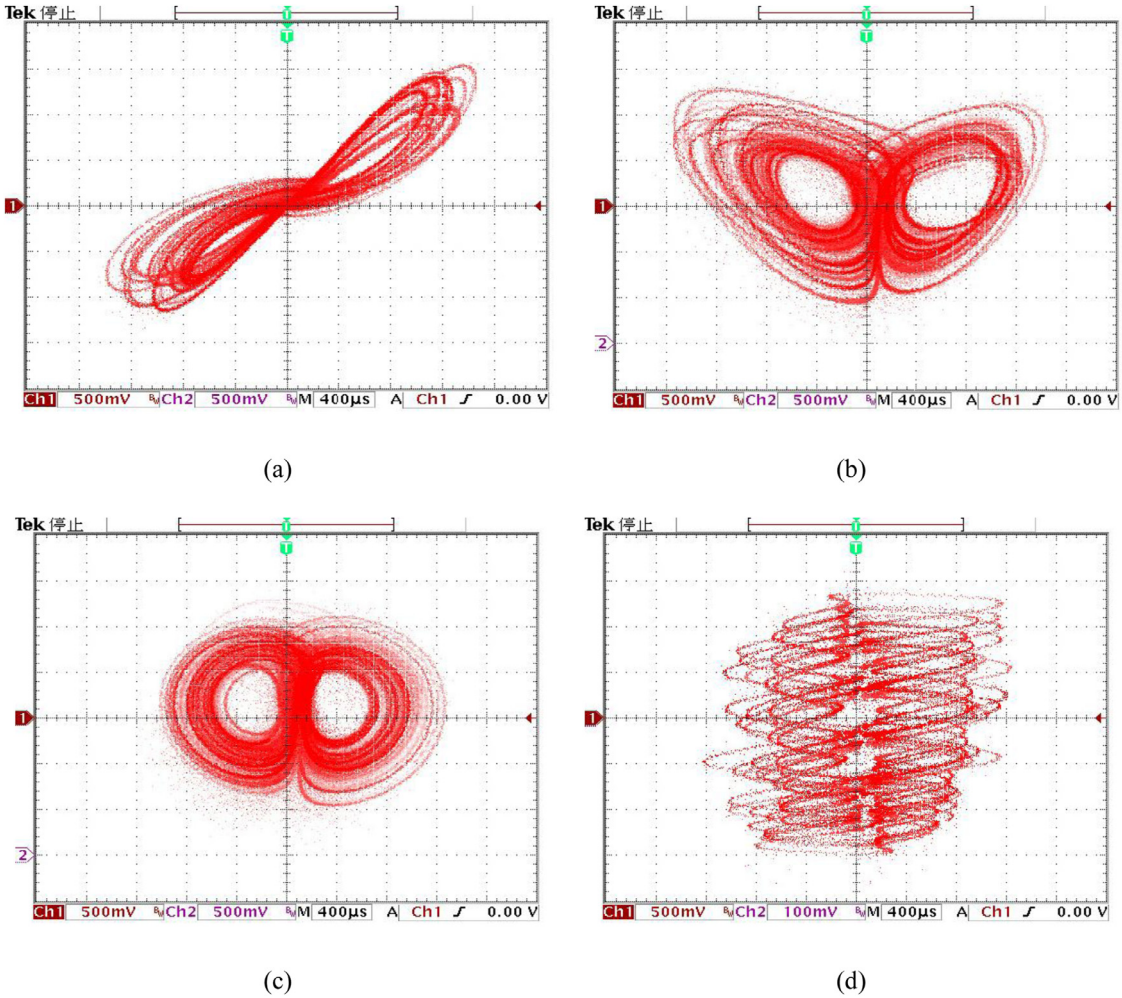
Tek 停止
Ch1 500mV Ch2 500mV M 400µs A Ch1 0.00 V
(a)
Tek 停止
Ch1 500mV Ch2 500mV M 400µs A Ch1 0.00 V
(b)
Tek 停止
Ch1 500mV Ch2 500mV M 400µs A Ch1 0.00 V
(c)
Tek 停止
Ch1 500mV Ch2 100mV M 400µs A Ch1 0.00 V
(d)

Fig. 10. Experimentally measured phase portraits of the hyperchaotic attractor of the memristive hyperchaotic system (6) with no equilibria: (a) in the v_x-v_y plane, (b) in the v_x-v_z plane, (c) in the v_y-v_z plane, and (d) in the v_y-v_w plane.

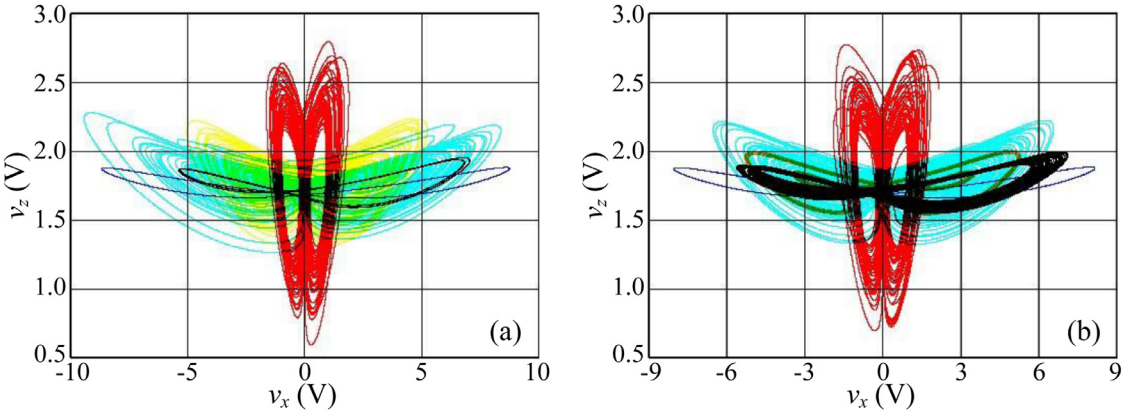

Fig. 11. PSIM simulated phase portraits of coexisting infinitely many hidden attractors: projections of different attractors on the v_x-v_z plane for different memristor initial conditions, (a) positive memristor initial conditions, $v_w(0) = 0.5$ V, (red), hyperchaos; $v_w(0) = 1.25$ V (yellow), hyperchaos; $v_w(0) = 2$ V (cyan), chaos; $v_w(0) = 2.45$ V (black), asymmetrical limit cycle; $v_w(0) = 3.5$ V (blue), symmetrical limit cycle; (b) negative memristor initial conditions, $v_w(0) = -1$ V (red), hyperchaos; $v_w(0) = -1.5$ V (cyan), chaos; $v_w(0) = -1.75$ V (yellow), chaos; $v_w(0) = -2.5$ V (black), intermittent chaos; $v_w(0) = -3.25$ V (blue), symmetrical limit cycle. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Based on the circuit simulation model in Fig. 9, the memristive hyperchaotic system is easily made in hardware level on a breadboard. The operational amplifiers OP07CP and analogue multipliers AD633JNZ with ± 15 V power supplies are utilized. All resistors and capacitors are selected as precision potentiometers and monolithic ceramic capacitors respectively. By Tektronix TDS 3034C digital oscilloscope, the phase portraits of the memristive hyperchaotic system in four different planes are experimentally measured, as shown in Fig. 10. Obviously, experimental measurements are consistent with numerical simulations of Fig. 2.

When different memristor initial conditions are considered, the interesting and striking phenomenon of infinitely many hidden attractors' behavior can be captured by PSIM circuit simulations, as shown in Fig. 11, where Fig. 11(a) reveals five kinds of hidden attractors projected on the v_x-v_z plane with five different positive memristor initial conditions; whereas Fig. 11(b) exhibits five kinds of hidden attractors projected on the v_x-v_z plane with five different negative memristor initial conditions. PSIM circuit simulations well validate the numerical simulations of Fig. 7(a) and (c), demonstrating the coexistence of infinitely many hidden attractors in the memristive hyperchaotic system.

6. Conclusions

In this paper, we established a novel memristive hyperchaotic system with no equilibrium based on the newly proposed circuit realization scheme, and investigated the phenomenon of extreme multistability with hidden oscillation that reveals the coexistence of infinitely many hidden attractors in the proposed memristive hyperchaotic system. These results verified by hardware experiments and PSIM circuit simulations indicate that the emergence of hidden extreme multistability in the memristive hyperchaotic system is initiated by the memristor initial condition-dependent dynamical behavior of the memristive hyperchaotic system. Considering that the memristor initial condition with different desired values is hardly assigned, the unusual and striking dynamical behavior of infinitely many hidden attractors emerged in the memristive hyperchaotic system is difficultly captured in hardware experiments. To solve this question could be subject of further research.

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